# Vieta’s Law

## Notes:

* For finding useful information from the coefficients of polynomials
* Easily derived from multiplying the linear factors of the polynomial
	+ If r is a root then f(x) is divisible by (x-r)
	+ If f(x) has n degrees then it has n complex roots
* For quadratic multiply (x-r1)(x-r2) and regroup to see coefficients, can also use quadratic formula
* Quadratic: ax2 + bx +c
	+ r1 + r2 = -b/a
	+ r1r2 = c/a
	+ What are the sum and product of the roots of the quadratic equation

x2 - 12x + 24 = 0 ?

* + What are the sum and product of the roots of the quadratic equation

3x2 + 15x - 18 = 0 ?

* For cubic just multiply out (x-r1)(x-r2)(x-r3) where the ra are roots of the cubic and look at the coefficients
* Cubic : ax3 + bx2 + cx +d
	+ r1+r2+r3 = -b/a
	+ r1r2 +r2r3 +r1r3 = c/a
	+ r1r2r3 = -d/a
	+ What are single sum, double sum, and product of the roots of the cubic equation

x3 + 4x2 − 10x − 18 = 0 ?

* Higher Degree:
	+ Same idea, for single sums, -b/a
	+ For two roots multiplied together sums, c/a
	+ Etc.
	+ Alternate plus and minus
* You can also manipulate the sums/products:
* Ex. 1:

If α and β are the roots of the equation 2x2 - 7x + 4 = 0, what is the value of 1/α + 1/β ?

* Ex. 2:

If g and h are the roots of the equation: x2 - 6x + 6 = 0
What is the value of: g2 + h2 ?

# Simon’s Favorite Factoring Trick

Notes:

* Useful algebraic manipulation technique
* General representation:

xy + ax + by = d

x(y + a) + b(y + a) = d + ab

(x + b)(y + a) = d + ab

* Ex. 1:

Factor 4xy + 6x + 10y.

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# Practice Questions

**2000 AMC 12 Problems/Problem 6**

Two different prime numbers between  and  are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?



**1987 AIME Problems/Problem 5**

Find  if  and  are [integers](https://artofproblemsolving.com/wiki/index.php/Integer) such that .

**2011 JBMO Problems/Problem 2**

Find all primes  such that there exist positive integers  that satisfy .

**2005 AMC 12A Problems/Problem 9**

There are two values of  for which the equation  has only one solution for . What is the sum of these values of ?



**1952 AHSME Problems/Problem 23**

If  has roots which are numerically equal but of opposite signs, the value of  must be:



**2008 AIME II Problems/Problem 7**

Let , , and  be the three roots of the equation![\[8x^3 + 1001x + 2008 = 0.\]]()Find .

**2009 AMC 12A Problems/Problem 16**

A circle with center  is tangent to the positive  and -axes and externally tangent to the circle centered at  with radius . What is the sum of all possible radii of the circle with center ?



**2009 AMC 12A Problems/Problem 17**

Let  and  be two different infinite geometric series of positive numbers with the same first term. The sum of the first series is , and the sum of the second series is . What is ?

**2007 Alabama ARML TST Problems/Problem 12**

If  and , then evaluate

![\[\dfrac{1}{1+w}+\dfrac{1}{1+w^2}+\dfrac{1}{1+w^3}+\cdots +\dfrac{1}{1+w^{2007}}.\]]()

Express your answer as a fraction in lowest terms.