# Vieta’s Law

## Notes:

* For finding useful information from the coefficients of polynomials
* Easily derived from multiplying the linear factors of the polynomial
  + If r is a root then f(x) is divisible by (x-r)
  + If f(x) has n degrees then it has n complex roots
* For quadratic multiply (x-r1)(x-r2) and regroup to see coefficients, can also use quadratic formula
* Quadratic: ax2 + bx +c
  + r1 + r2 = -b/a
  + r1r2 = c/a
  + What are the sum and product of the roots of the quadratic equation

x2 - 12x + 24 = 0 ?

* + What are the sum and product of the roots of the quadratic equation

3x2 + 15x - 18 = 0 ?

* For cubic just multiply out (x-r1)(x-r2)(x-r3) where the ra are roots of the cubic and look at the coefficients
* Cubic : ax3 + bx2 + cx +d
  + r1+r2+r3 = -b/a
  + r1r2 +r2r3 +r1r3 = c/a
  + r1r2r3 = -d/a
  + What are single sum, double sum, and product of the roots of the cubic equation

x3 + 4x2 − 10x − 18 = 0 ?

* Higher Degree:
  + Same idea, for single sums, -b/a
  + For two roots multiplied together sums, c/a
  + Etc.
  + Alternate plus and minus
* You can also manipulate the sums/products:
* Ex. 1:

If α and β are the roots of the equation 2x2 - 7x + 4 = 0, what is the value of 1/α + 1/β ?

* Ex. 2:

If g and h are the roots of the equation: x2 - 6x + 6 = 0   
What is the value of: g2 + h2 ?

# Simon’s Favorite Factoring Trick

Notes:

* Useful algebraic manipulation technique
* General representation:

xy + ax + by = d

x(y + a) + b(y + a) = d + ab

(x + b)(y + a) = d + ab

* Ex. 1:

Factor 4xy + 6x + 10y.

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# Practice Questions

**2000 AMC 12 Problems/Problem 6**

Two different prime numbers between $4$ and $18$ are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

$\mathrm{(A)}\ 21 \qquad\mathrm{(B)}\ 60 \qquad\mathrm{(C)}\ 119 \qquad\mathrm{(D)}\ 180 \qquad\mathrm{(E)}\ 231$

**1987 AIME Problems/Problem 5**

Find $3x^2 y^2$ if $x$ and $y$ are [integers](https://artofproblemsolving.com/wiki/index.php/Integer) such that $y^2 + 3x^2 y^2 = 30x^2 + 517$.

**2011 JBMO Problems/Problem 2**

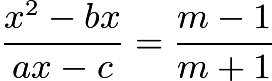
Find all primes $p$ such that there exist positive integers $x,y$ that satisfy $x(y^2-p)+y(x^2-p)=5p$.

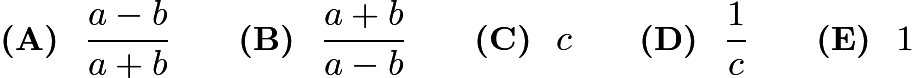
**2005 AMC 12A Problems/Problem 9**

There are two values of $a$ for which the equation $4x^2 + ax + 8x + 9 = 0$ has only one solution for $x$. What is the sum of these values of $a$?

$(\mathrm {A}) \ -16 \qquad (\mathrm {B}) \ -8 \qquad (\mathrm {C})\ 0 \qquad (\mathrm {D}) \ 8 \qquad (\mathrm {E})\ 20$

**1952 AHSME Problems/Problem 23**

If  has roots which are numerically equal but of opposite signs, the value of $m$ must be:



**2008 AIME II Problems/Problem 7**

Let $r$, $s$, and $t$ be the three roots of the equation\[8x^3 + 1001x + 2008 = 0.\]Find $(r + s)^3 + (s + t)^3 + (t + r)^3$.

**2009 AMC 12A Problems/Problem 16**

A circle with center $C$ is tangent to the positive $x$ and $y$-axes and externally tangent to the circle centered at $(3,0)$ with radius $1$. What is the sum of all possible radii of the circle with center $C$?

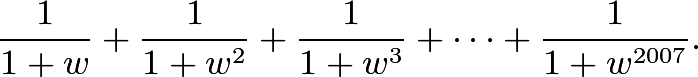
$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

**2009 AMC 12A Problems/Problem 17**

Let $a + ar_1 + ar_1^2 + ar_1^3 + \cdots$ and $a + ar_2 + ar_2^2 + ar_2^3 + \cdots$ be two different infinite geometric series of positive numbers with the same first term. The sum of the first series is $r_1$, and the sum of the second series is $r_2$. What is $r_1 + r_2$?

**2007 Alabama ARML TST Problems/Problem 12**

If $w^{2007}=1$ and $w\neq 1$, then evaluate



Express your answer as a fraction in lowest terms.